REVIEWS

Viscous Flow Theory, II—Turbulent Flow, by SHIH-I PAI. Princeton: D. Van Nostrand Co., 1957. 277 pp. $6.75 or 48*s*.

In a time which most people would find barely adequate for the writing of one book, Professor Pai has published no fewer than three on different branches of fluid mechanics. Beginning with *Fluid Dynamics of Jets* (1954), he followed up with *Viscous Flow Theory, I—Laminar Flow* (1956), and now we have this new work on turbulent flow. The two parts of the treatise on viscous flow theory are really separate books, since there is not much in common, pedagogically speaking, between laminar and turbulent flow; indeed, there is not even much of a case for the use of a common title in view of the relatively unimportant part played by viscosity in most cases of turbulent flow. It is no mean achievement for a man to find the determination and knowledge needed for three books in rapid succession, and it might be supposed that they have a purpose which is obvious. I have not in fact been able to discern clearly the purpose served by the present book or to classify it as one of the common types. I do not think that it can be called a research monograph on turbulent flow; the author does not reveal important new advances, and his book does not have the physical insight and critical appraisal that would be needed for it to say anything new to those in touch with the subject. Nor can it be said to be an exposition for students; the material is too raw and incomplete, and includes too many unrelated items of current research which are of unknown significance, for the book to be suitable for teaching purposes. There is undoubtedly a need for a textbook on turbulent flow, but the present work does not have the regard for fundamentals and for logical coherence that such a text should have. Despite the claim made on the dust-jacket (with characteristic punctuation and spelling) that " Engineers, research scientists as well as advanced students will find this complete treatment of turbulent flow an indispensible source of fresh information ", it seems to me to lack both the indispensability of a good textbook and the freshness of a good research monograph.

So far as technique of writing is concerned, the author has acted simply as a reporter, absorbing large masses of paper and regurgitating it in a uniform style in the manner of a journalist. The present book, like a good newspaper, can fairly be said to be comprehensive in scope, up to the minute, impartial, and discursive. It also has the defects of some journalistic reporting—a limited respect for the truth, haste in composition and printing, some irresponsibility, and an apparent reluctance to think about the material for longer than it takes to read it and write it down in different (not always!) words. Professor Pai does have a genuine flair for reporting scientific developments; as many visitors to the University of Maryland have found, he makes a creditable job of writing up their lectures for later reproduction and distribution by the University. This kind of writing requires mental agility and elasticity to an unusual degree. But though the rapid reporting of many different scientific subjects demands considerable skill, the job encourages superficiality and this is unfortunately the hall-mark of the book under review. Passing lightly from one snippet of news to another, the text always stops short of the point of illumination and fails completely to provide a clear picture of the basic mechanics of turbulent flow.

The scope of the book is wide, and embraces all aspects of turbulent motion; the classical cases of turbulent shear flow, mixing-length theories, boundary layers with pressure gradients, compressible turbulent flow, turbulent diffusion, decaying isotropic turbulence, spectrum theory, Kolmogoroff's similarity theory, and even turbulence in an electrically conducting fluid, all find some kind of place in the book. The treatments of these topics are not thorough (nor could they be in a book of this size), and a more cautious author would have hesitated to try to compress a whole subject like turbulent diffusion into 18 pages. But granted that it is useful to say something about everything within the space of one book, there is an acute problem of selection of material to be faced. The pieces of work which Professor Pai chooses for inclusion show him to be attracted strongly by those having a mathematical form and hardly at all by physical thought; for instance, the book includes fairly full accounts of Reichardt's "inductive theory of turbulence" (pp. 37, 129), the method of determining profile drag from wake measurements (p. 135), Goldstein's hyperbolic diffusion equation (p. 186), correlation and spectrum tensor relations (pp. 214–230), and Chandrasekhar's 'propagation' of density fluctuations (p. 256), but on the other hand makes no mention whatever of what is perhaps the most fundamental (and practically useful) formula in the theory of turbulence, namely, that the rate of dissipation of energy is of the same order of magnitude as u^3/l, where u and l are representative velocity and length scales of the turbulence. The more straightforwardly mathematical investigations in any subject are undoubtedly the easiest to comprehend, and are often seized upon with relief by students; for that very reason, it is desirable always to explain their significance, the assumptions on which they are based, their implications, and their agreement with observation. I am sure Professor Pai would agree with that precept, but I do not think he has done much about it; he says (p. 209), "... we have no general methods for solving such nonlinear equations. At the present time, we try to obtain an understanding of the mechanism of turbulence by studying experimental results ...", but in fact he does not provide a single diagram or table of observations relating to homogeneous turbulence even though about half the book is concerned with that case. His bias toward highly theoretical enquiries sometimes leads him into rather pretentious and irrelevant generalities, as for instance when (ch. 8) he drags in phase space and Liouville's theorem and other concepts of statistical mechanics.

The book falls into two non-overlapping and non-interacting parts, the first being concerned with "semi-empirical theories of turbulence" and the standard cases of turbulent shear flow, and the second with the "statistical theory of turbulence" and isotropic turbulence in particular. This dichotomous approach to the subject of turbulent flow was remarked in a recent review of *Momentum Transfer in Fluids* by Corcoran, Opfell and Sage (*J. Fluid Mech.* **2**, 1957, 204); indeed, had Professor Pai's book appeared a little earlier, it would have been convenient to review them together since they have a good deal in common quite apart from their concern with turbulent flow. The division is made quite sharply here, the mean velocity distribution in jets, wakes, etc. being regarded as belonging to the "semi-empirical theories" and velocity fluctuations in those same flows as belonging to the "statistical theories". Unless someone soon writes a book which integrates the old and the new, the means and the fluctuations, the "semi-empirical" and the "statistical", we shall have a generation of students who believe that there must be some basis in nature for this dual approach and that jets and wakes really do have two faces. Mixing-length theories and the statistical terminology represent the contributions of two separate periods of research (both of them now past), and we ought by now to be able to make up our minds about what is good in each and to allow what is not good to be quietly forgotten.

The first part of the book on the "semi-empirical theories" follows a familiar and appropriate pattern. There is first a chapter introducing the momentum and vorticity transfer theories and hypotheses about the mixing length, and containing a rather perfunctory account of the facts that they are intended to explain. The two key empirical laws concerning the mean velocity are the universality of the distributions in the central region of a channel and near a rigid plane boundary. These two laws deserve prominence, as any number of writers have pointed out, but Professor Pai passes over them lightly and does not even distinguish them clearly (p. 28). The next five chapters consider in turn the cases of channel flow, a flat plate without and with a pressure gradient, a compressible boundary layer, and what the book calls "turbulent jet mixing regions and wakes". I cannot see here any improvement on the corresponding parts of the books by Schlichting (1950) and Goldstein (1938), any advantage conferred by the date of the present book being more than balanced by the greater perspicacity of these earlier writers. The author perpetuates the misleading half-truth that near a wall "there is a laminar sub-layer in which the flow behaves as a laminar flow" (p. 40) and that the relative velocity fluctuations fall to zero at the wall (p. 242). He makes the calculation of turbulent boundary layer parameters seem even more like a set of arbitrary rules than it usually does, and even though the chapter on the compressible boundary layer is a brave attempt to be up-to-date, he has not succeeded in giving the various contributions to that problem any order and coherence. In his description of free turbulent flows, the wonderful phenomenon of the sharp transition from rotational to irrotational flow at an irregular fluctuating boundary is mentioned only once (p. 75) and it is there referred to as a postulate.

The second part of the book, on the "statistical theory", is evidently more congenial to the author, and he makes full use of the greater opportunities for presenting mathematical analysis. He has had the useful idea of including an account of elementary probability theory, and he has taken particular trouble over the account of harmonic analysis of random functions. He uses the box device of first imagining the random function to be zero outside some distant boundary in order that the Fourier coefficients should exist, and of allowing the boundary to recede to infinity at an appropriate stage of the analysis. The approach has the advantage of directness, although in Professor Pai's hands it is rather lengthy (and he surely unnerves his readers when he says (p. 217)—wrongly—that the shape of the box in three dimensions could affect the results after the limit is taken). He makes a number of portentous remarks about ergodicity in general and about the relation between theoretical and experimental averages in turbulence, but he fails to say clearly (indeed it is not evident that he knows it) that stationarity in the variable concerned in a necessary, and usually in practice a sufficient, condition for equality of the ensemble average and an integral average. The whole treatment of stationary random functions is poor, and there is confusion (pp. 161–164) between the various values taken by a random variable and the various values taken by a random function of *t* at different values of *t* in one realization of the function. There are several gaffes, as when he states that an auto-correlation function is positive definite (p. 213) and that non-normality of the probability distributions of velocity derivatives indicates that "they are not entirely random in nature".

After these mathematical preliminaries, most of the standard analysis of homogeneous turbulence is presented. Provided no physical arguments are involved, Professor Pai gets by without much error, although the selection of material is curious and undiscriminating. Where physical justification of the analysis is needed, as in the account of Kolmogoroff's similarity theory, and in the brief excursion into turbulence in a conducting fluid, the text is quite inadequate. The introduction to chapter 13 reflects confusion between the various wave-number ranges of the spectrum and the various stages of decay; it may have been this confusion which led Professor Pai into the persistent error that inertia forces have no effect on the smallest eddies (p. 234) and that as a consequence the energy spectrum has a Gaussian form at large wave-numbers (p. 238). Nowhere does one hear about the law of decay of turbulent energy and the spectral distribution that are actually observed in real wind tunnels. As for the chapter on turbulent diffusion, I can say only that evidently Professor Pai and I have quite different views about what are the real problems involved.

I am sure that my general opinion of this book has already revealed itself sufficiently. Like the book by Corcoran, Opfell and Sage, I do not think it can be recommended, either as a text for postgraduate students or as a source of information or enlightenment to more senior people. Again like this earlier book, it is written in rudimentary English, and with little regard for the rules of grammar and punctuation.

As a less sober postscript to this review, I, like the reviewer of Part I of the present work (*J. Fluid Mech.* **2**, 1957, 515), was struck by Professor Pai's diverting habit of incorporating slices of other people's papers in his book. This is perhaps carrying the reporting technique a little too far. I was able to recognize three whole paragraphs and some odd sentences from two of my own papers in the chapter on diffusion, and more of my own paragraphs in other chapters (with no hint that a direct quotation was being given and often without reference to the source). He even uses this strange technique on his own writing, and on page 146 reproduces what he has given on page 10. How many other writers are represented in this book? As a method of constructing a book it is so absurd that one cannot be indignant at being copied in this way. Scissors and paste may be useful accessories at a lower level of writing, but at this level it is impossible to integrate existing pieces of prose into a continuous reasoned narrative. An author cannot avoid mastering a subject and rewriting it himself.

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Magnetohydrodynamics, by T. G. COWLING. New York: Interscience Publishers, Inc., 1957. 115 pp. $3.50.

The motion of a fluid or a gaseous medium possessing electrical conductivity in the presence of a magnetic field has become of great importance in the last decade. The theoretical treatment is variously designated as 'magneto-hydrodynamics' or 'magneto-gasdynamics', or more generally 'magneto-fluid dynamics', and sometimes, less characteristically, as 'hydromagnetics'. Although a few experimental investigations on the influence of magnetic forces on the motion of a conducting liquid date from the previous century and the discovery of the magnetic field of sunspots was made in the first decade of the present century, it has taken some time for a wider interest to develop. One feature leading to this wider interest seems to have been the discovery of interstellar polarization of light, which for its explanation required the assumption of interstellar magnetic fields of wide extension. Problems connected with cosmic radiation led to the same conclusion. Once the presence of such fields was recognized, many other problems called for investigation. Almost at the same time the wide application of shock waves for the investigation of high temperature gas motion with its consequent ionization led to the problem whether such flows could be influenced by means of magnetic fields; research on the behaviour of magnetically driven plasmas arose out of this. Perhaps two landmarks in the development of the subject were the first Symposium on Cosmical Gas Dynamics (Paris, 1949, later followed by meetings at Cambridge, England, 1953, and Cambridge, Massachusetts, 1957); and H. Alfvén's book *Cosmical Electrodynamics*, published in 1950. From 1950 onward papers on astrophysical problems and on results of experimental work have come out in rapid succession; the number of conferences is multiplying, and now technical applications are in view. Thus there is a need for a treatise which may help interested scientists and engineers to become acquainted with the many aspects of magneto-fluid dynamics.

Professor Cowling's book is not intended as a treatise, but as a condensed survey. The topics treated are briefly as follows. Chapter 1 gives the basic equations and general ideas, stressing the distinction between cases where the electrical resistance of the medium is a determining influence, and those cases where the conductivity of the medium can be considered as being practically infinite. The meanings of the magnetic Reynolds number and of other non-dimensional quantities are pointed out. Since one of the factors occurring in the magnetic Reynolds number is the length scale of the field, the case of 'infinite conductivity' is of great importance in astrophysical and geophysical systems with their large dimensions, the decay times for electric current systems being here of cosmical duration. As an illustration of a different case a simple treatment is given of laminar flow under the combined influence of ordinary viscosity and what is sometimes termed 'magnetic viscosity'. Chapter 2 is about 'magnetostatics' and considers equilibrium states in which the mechanical forces are balanced by magnetic stresses. Applications are made to some astronomical problems (sunspots, streamers and filaments, and spiral arms of the galaxy). Chapter 3 treats wave motion, mainly for an incompressible fluid where the velocity of propagation is wholly determined by magnetic effects (the so-called 'Alfvén waves', connected with the effective tension along the lines of magnetic flux). Again some astrophysical applications are mentioned. Chapter 4 considers the influence of magnetic forces on the stability or instability of fluid motion, with examples taken from cases of laminar (incompressible) flow and from problems of convection. Chapter 5 is entitled 'Dynamo theories' and considers the question whether fluid or gas motions in the presence of an originally weak magnetic field can lead to an increase of the field strength, in the way in which this occurs in a self-exciting dynamo. Elsasser's, Bullard's, and Parker's theories of the origin of the magnetic field of the earth are considered and criticized. The chapter also includes a brief discussion of turbulent fields of motion and of the distribution of energy between turbulent motion and magnetic field. Finally chapter 6 gives attention to the effects of molecular structure, as they appear in ionized gases. Each chapter is concluded with a short list of the most important and relevant publications.

From this summary it will be seen that Cowling's book is of value to all who wish to become acquainted with various topics of magneto-fluid dynamics. But on the other hand, considered from the point of view of the needs of this moment, the book leaves many things to be desired. Although it is evident that in the short space set for the Interscience Tracts on Physics and Astronomy (of which this book is one) there is no room for the writing of a textbook, one has the feeling that in preparing it the author has limited himself mostly to a critical discussion of a series of selected topics, in order to point out where theories are incomplete. Professor Cowling, who is an authority in the field of gas motion under the influence of magnetic fields, of course has full right to do so, but as a result of this the reader is not greatly stimulated to enter into the subject and to attempt to work out problems for himself. It is clear moreover that the emphasis in the selection is on the geophysical and astronomical applications, as Professor Cowling himself observes in his preface; although he adds the hope that the publication of the book may expedite developments also in the engineering aspects, one cannot say that he is giving much help in this respect.

Chapter 6, for instance, on ionized gases, indeed gives the relevant equations and puts them into various forms, and these forms are discussed from a number of points of view, but at the end the reader still may ask: where do I stand? How can I apply this to a particular problem? Mean collision times are introduced, but there is no indication concerning the methods in use for calculating or estimating them (although some of these methods are considered in *The Physics of Fully Ionized Gases*, by L. Spitzer, Jr., which appeared earlier in the same series of tracts and which may in a sense be regarded as a companion volume). In some of the other chapters of Professor Cowling's book, in the treatment of problems concerning the sun, the magnetic field of the galaxy, the magnetic field of the earth, and turbulence theory, the ultimate impression obtained is rather a negative one; it looks as if almost nothing fruitful can be done in these matters.

Problems concerning the behaviour of acoustic waves, and still less problems referring to shock waves in a magnetic field, have not been considered at all. Here a great many cases are awaiting adequate treatment and it would have been useful if at least some indications had been given. There are, for instance, three-dimensional problems concerning the propagation of shock waves approaching a magnetic field (in experimental situations one always has to do with non-uniform fields); there is also the problem of a shock wave in a cold gas subjected to a magnetic field, the electrical conductivity appearing only behind the wave front as a consequence of the high temperatures generated there. There is the problem of stationary supersonic flow of an ionizable gas around a magnetized body. Boundary layer flow under conditions leading to ionization and with magnetic field influencing the motion likewise calls for attention. Still another group of problems refers to plasmas driven by magnetic fields; this is under investigation at present on account of the very high temperatures which can be produced in them. The magnetic field due to the plasma current itself, as well as fields derived from outside sources, are both of importance in guiding the motion of the plasma. The question of the stability of the plasma current involves the so-called 'pinch effect' (the lateral contraction of a plasma current as a consequence of its own magnetic field); Professor Cowling does make two short references to this effect (pp. 27, 29) without, however, making any reference to the intriguing questions connected with it. (Some discussion of the 'pinch effect' is to be found in Spitzer's book.)

Certainly we are grateful for the material Professor Cowling has put so lucidly in 115 pages. The book seems well adapted to the needs of astrophysicists and geophysicists who wish to obtain a rapid view of the present state of the application of magneto-fluid dynamics to their field. But one who comes from the aerodynamical side still feels the need for a work of more comprehensive character, which can serve as a guide for those who wish to develop their ability to work in this interesting domain.

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